SOLUTIONS TO EXERCISE 2

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5.1

(a). True. The t test is based on variables with a normal distribution. Since the estimators of β_1 and β_2 are linear combinations of the error u_i, which is assumed to be normally distributed under CLRM, these estimators are also normally distributed.

(b). True. So long as $E(u_i) = 0$, the OLS estimators are unbiased. No probabilistic assumptions are required to establish unbiasedness.

(c). True. In this case the Eq. (1) in App. 3A, Sec. 3A.1, will be absent. This topic is discussed more fully in Chap. 6, Sec. 6.1.

(d). True. The p value is the smallest level of significance at which the null hypothesis can be rejected. The terms level of significance and size of the test are synonymous.

(e). True. This follows from Eq. (1) of App. 3A, Sec. 3A.1.

(f). False. All we can say is that the data at hand does not permit us to reject the null hypothesis.

(g). False. A larger 2 s may be counterbalanced by a larger $\sum x_i^2$. It is only if the latter is held constant, the statement can be true.

(h). False. The conditional mean of a random variable depends on the values taken by another (conditioning) variable. Only if the two variables are independent, that the conditional and unconditional means can be the same.

(i). True. This is obvious from Eq. (3.1.7).

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(j). True. Refer of Eq. (3.5.2). If X has no influence on Y, $\hat{\beta}_2$ will be zero, in which case $\sum y_i^2 = \sum \hat{u}_i^2$.

5.5

(a). Use the t test to test the hypothesis that the true slope coefficient is one. That obtains:

$$t = \frac{\hat{\beta}_2 - 1}{se\left(\hat{\beta}_2\right)} = \frac{1.0598 - 1}{0.0728} = 0.821$$

For 238 *df* this t value is not significant even at $\alpha = 10\%$. The conclusion is that over the sample period, IBM was not a volatile security.

(b). Since $t = \frac{0.7264}{0.3001} = 2.4205$, which is significant at the two percent level of significance. But it has little economic meaning. Literally interpreted, the intercept value of about 0.73 means that even if the market portfolio has zero return, the security's return is 0.73 percent.

5.8

(a). There is a positive association in the LFPR in 1972 and 1968, which is not surprising in view of the fact since WW II there has been a steady increase in the LFPR of women.

(b). Use the one-tail t test. $t = (0.6560 - 1)/0.1961 = -1.7542$ For 17 *df*, the one-tailed t value at α =5% is 1.740. At this level of significance, we can reject the null hypothesis that the true slope coefficient is 1 or greater.

(c). The mean LFPR is : $0.2033 + 0.6560(0.58) \approx 0.5838$. To establish a 95% confidence interval for this forecast value, use the formula: 0.5838 ± 2.11 (se of the mean forecast value), where 2.11 is the 5% critical t value for 17 *df*. To get the standard error of the forecast value, use Eq. (5.10.2). But note that since the authors do not give the mean value of the LFPR of women in 1968, we cannot compute this standard error.

(d). Without the actual data, we will not be able to answer this question because we need the values of the residuals to plot them and obtain the Normal Probability Plot or to compute the value of the Jarque-Bera test.

5.14

(a). None appears to be better than the others. All statistical results are very similar. Each slope coefficient is statistically significant at the 99% level of confidence.

(b). The consistently high r^2s cannot be used in deciding which monetary aggregate is best. However, this does not suggest that it makes no difference which equation to use.

(c). One cannot tell from the regression results. But lately the Fed seems to be targeting the M2 measure.

5.15

Write the indifference curve model as:

$$Y_i = \beta_1 \left(\frac{1}{X_i} \right) + \beta_2 + u_i$$

Note that now β_1 becomes the slope parameter and β_2 the intercept. But this is still a linear regression model, as the parameters are linear (more on this in Ch.6). The regression results are as follows:

$$\begin{aligned} \hat{Y}_i &= 3.2827 \left(\frac{1}{X_i} \right) + 1.1009 \\ se &= (1.2599) \qquad\quad (0.6817) \\ r^2 &= 0.6935 \end{aligned}$$

The "slope" coefficient is statistically significant at the 92% confidence coefficient. The marginal rate of substitution (MRS) of Y for X is:

$$\frac{\partial}{\partial X} Y = -3.2827 \left(\frac{1}{X_i^2} \right)$$

6.1

True. Note that the usual OLS formula to estimate the intercept is $\hat{\beta}_1$ =(mean of the regressand - $\hat{\beta}_2$ mean of the regressor). But when Y and X are in deviation form, their mean values are always zero. Hence in this case the estimated intercept is also zero.

6.2

(a & b). In the first equation an intercept term is included. Since the intercept in the first model is not statistically significant, say at the 5% level, it may be dropped from the model.

(c). For each model, a one percentage point increase in the monthly market rate of return lead on average to about 0.76 percentage point increase in the monthly rate of return on Texaco common stock over the sample period.

(d). As discussed in the chapter, this model represents the characteristic line of investment theory. In the present case the model relates the monthly return on the Texaco stock to the monthly return on the market, as represented by a broad market index.

(e). No, the two r^2s are not comparable. The r^2 of the interceptless model is the raw r^2.

(f). Since we have a reasonably large sample, we could use the Jarque-Bera test of normality. The JB statistic for the two models is about the same, namely, 1.12 and the p value of obtaining such a JB value is about 0.57. Hence do not reject the hypothesis that the error terms follow a normal distribution.

(g). As per Theil's remark discussed in the chapter, if the intercept term is absent from the model, then running the regression through the origin will give more efficient estimate of the slope coefficient, which it does in the present case.

6.7

Equation (6.6.8) is a growth model, whereas (6.6.10) is a linear trend model. The former gives the relative change in the regressand, whereas the latter gives the absolute change. For comparative purposes it is the relative change that may be more meaningful.

6.13

(a). For every tenth of a unit increase (0.10) in the Gini coefficient, we would expect to see a 3.32 unit increase in a countryâĂŹs sociopolitical instability index. Therefore, as the Gini coefficient gets higher, or a countryâĂŹs income inequality gets larger, a country becomes less socio-politically stable.

(b). To see this difference, simply assess what happens if the Gini coefficient increases by 0.3. So, $33.2(0.3) = 9.96$, indicating an increase of 9.96 in the SPI.

(c). Using the standard t test,$t = 33.2/11.8 = 2.8136$ for testing the null hypothesis that the slope coefficient is 0. For 38 degrees of freedom, the critical value from the table in Appendix D is somewhere between 2.021 and 2.042 (using a two-sided test), so the estimated slope is statistically significant at the 5% level.

(d). Based on the regression results, we can conclude that there is a positive relationship between higher income inequality and greater political instability, although we cannot make a causal statement about the relationship.

6.14

$$\begin{aligned} \frac{100}{100 - Y_i} &= 2.0675 + 16.2662\left(\frac{1}{X_i}\right) \\ se &= (0.1596) \quad (1.3232) \\ r^2 &= 0.9497 \end{aligned}$$

As X increases indefinitely, $100/(100 - Y)$ approaches the limiting value of 2.0675, which is to say that Y approaches the limiting value of about 51.6.