Applications with Bayesian Approach

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Outline

Clinical trial of recombinant human growth hormone (rhGH) for increasing muscle strength in the elderly(Daniels & Hogan 2008)

- The data come from a randomized clinical trial conducted to examine the effects of recombinant human growth hormone (rhGH) therapy for building and maintaining muscle strength in the elderly.
- The study enrolled 161 participants and randomized them to one of four treatment arms: placebo (P), growth hormone only (G), exercise plus placebo (EP), and exercise plus growth hormone (EG).
- Various muscle strength measures were recorded at baseline, 6 months, and 12 months.
- Roughly 75% of randomized individuals completed all 12 months of followup, and most of the dropout was thought to be related to the unobserved responses at the dropout times.

Treatment	s	n_s	Month 0	6	12
EG	1	12	58 (26)		
	2	4	57 (15)	68 (26)	
	3	22	78 (24)	90 (32)	88 (32)
	All	38	69 (25)	87 (32)	88 (32)
G	1	6	68 (17)		
	2	5	77 (33)	81 (42)	
	3	30	67 (22)	64 (21)	63 (20)
	All	41	69 (23)	66 (25)	63 (20)
EP	1	7	65 (32)		
	2	2	87 (52)	86 (51)	
	3	31	65 (24)	81 (25)	73 (21)
	All	40	66 (26)	82 (26)	73 (21)
P	1	8	66 (29)		
	2	5	53 (19)	62 (31)	
	3	28	67 (23)	62 (20)	63 (19)
	All	41	65 (24)	62 (22)	63 (19)

The pattern mixture model

- For the mixture model specification, the full-data response distribution is a mixture over the follow-up times, factored as

$$p(y_{\mathrm{mis}}, y_{\mathrm{obs}}, s|z) = p(y_{\mathrm{mis}}|y_{\mathrm{obs}}, s, z)p(y_{\mathrm{obs}}|s, z)p(s|z)$$

  where s is the pattern and this model allows **missing not at random**.

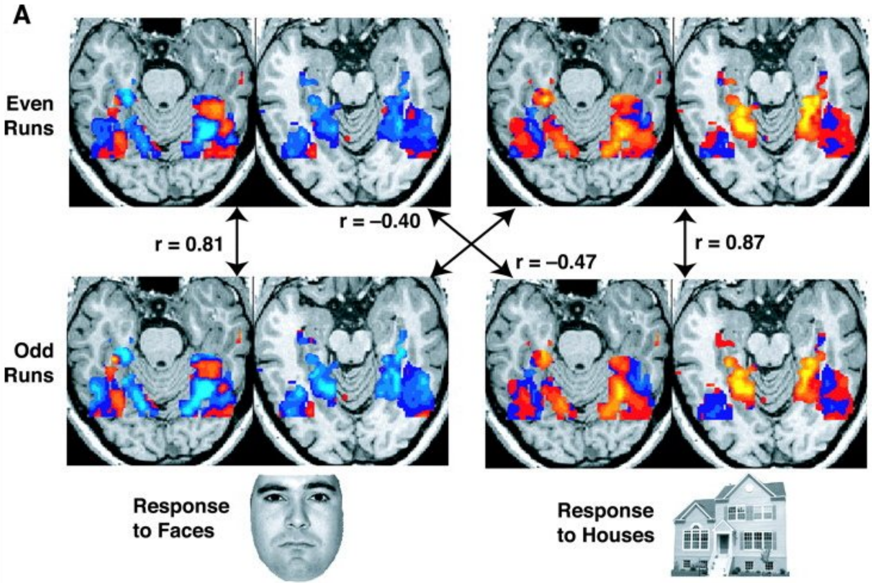
Table 10.1 *Growth hormone trial: posterior mean (s.d.) for quadriceps strength at each time point, stratified by treatment group. MVN = multivariate normal distribution assumed for joint distribution of responses; MAR = missing at random constraints; MNAR-1 =* $\boldsymbol{\Delta}$ *assumed common across treatment groups; MNAR-2 =* $\boldsymbol{\Delta}$ *assumed different by treatment group. Both MNAR analyses use uniform priors for* $\boldsymbol{\Delta}$ *bounded away from zero. See Section 10.2.8 for details.*

			Pattern Mixture Models		
Treatment	Month	MVN	MAR	MNAR-1	MNAR-2
EP	0	65 (4.2)	66 (4.6)	66 (4.6)	66 (4.6)
	6	81 (4.4)	82 (4.7)	80 (4.9)	80 (4.9)
	12	73 (3.7)	73 (4.0)	70 (4.3)	69 (4.6)
EG	0	69 (4.2)	69 (4.4)	69 (4.4)	69 (4.4)
	6	81 (6.0)	81 (6.4)	78 (6.8)	78 (6.8)
	12	78 (6.3)	78 (6.7)	73 (7.4)	72 (8.1)
Difference at 12 mos.		5.7 (7.3)	5.4 (7.8)	3.1 (8.2)	2.6 (9.3)

Multilevel linear modelling for FMRI group analysis using Bayesian inference (Woolrich et al. 2004)

- Functional magnetic resonance imaging (FMRI) studies often involve the acquisition of data from multiple sessions and/or multiple subjects.
- A hierarchical approach can be taken to modelling such data with a general linear model (GLM) at each level of the hierarchy introducing different random effects variance components.
- Inferring on these models is nontrivial with frequentist solutions being unavailable. A solution is to use a Bayesian framework.

FMRI data


A
Even
Runs
Odd
Runs
r = 0.81
r = –0.40
r = –0.47
r = 0.87
Response
to Faces
Response
to Houses

The two-level univariate GLM for FMRI

- Consider the familiar two-level univariate GLM for FMRI. For example, the model that in the first level deals with individual sessions for individual subjects, relating time series to activation, and in the second level deals with a group of subjects or sessions (or both), relating the combined individual activation estimates to some group parameter, such as mean activation level.
- The two-level model is

$$
\begin{aligned}
Y &= X\beta_k + e_k \\
\beta_k &= X_g \beta_g + e_g
\end{aligned}
$$

- Markov chain Monte Carlo (MCMC) is used to sample from the full joint posterior distribution

Neuroeconomics

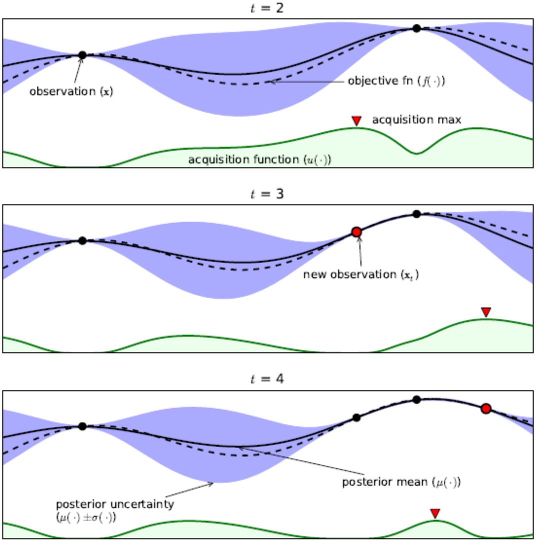
- Neuroeconomics is an interdisciplinary field that seeks to explain human decision making.
- It studies how economic behavior can shape our understanding of the brain, and how neuroscientific discoveries can constrain and guide models of economics
- There are several different techniques that can be utilized to understand the biological basis of economic behavior.
- Neural imaging is used in human subjects to determine which areas of the brain are most active during particular tasks. Some of these techniques, such as FMRI are best suited to giving detailed pictures of the brain which can give information about specific structures involved in a task (Volz et al. 2004).

Global Optimization

- An enormous body of scientific literature has been devoted to the problem of optimizing a nonlinear function $f(x)$ over a compact set A. In the realm of optimization, this problem is formulated concisely as follows:

$$max f(x)$$

- One typically assumes that the objective function $f(x)$ has a known mathematical representation, is convex, or is at least cheap to evaluate. Despite the influence of classical optimization on machine learning, many learning problems do not conform to these strong assumptions.


$t = 2$
observation ($\mathbf{x}$)
objective fn ($f(\cdot)$)
acquisition max
acquisition function ($u(\cdot)$)
$t = 3$
new observation ($\mathbf{x}_t$)
$t = 4$
posterior mean ($\mu(\cdot)$)
posterior uncertainty
($\mu(\cdot) \pm \sigma(\cdot)$)

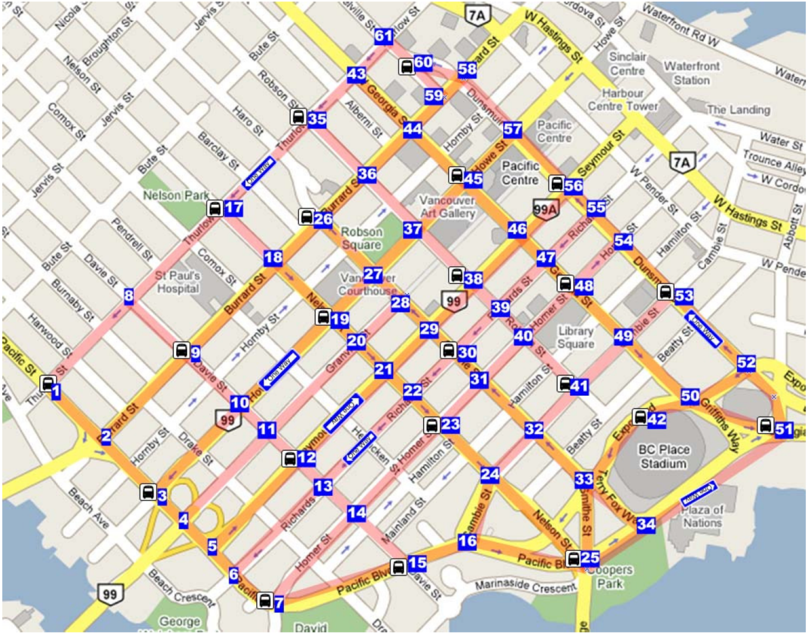
The Bayesian Optimization Approach(Brochu et al. 2010)

Algorithm 1 Bayesian Optimization

for $t = 1, 2, \dots$ **do**
  Find $\mathbf{x}_t$ by optimizing the acquisition function over the GP: $\mathbf{x}_t = \operatorname{argmax}_{\mathbf{x}} u(\mathbf{x}|\mathcal{D}_{1:t-1})$.
  Sample the objective function: $y_t = f(\mathbf{x}_t) + \varepsilon_t$.
  Augment the data $\mathcal{D}_{1:t} = \{\mathcal{D}_{1:t-1}, (\mathbf{x}_t, y_t)\}$ and update the GP.
end for

The Open Racing Car Simulator

- The Open Racing Car Simulator a 3D game engine that implements complex vehi- cle dynamics complete with manual and automatic transmission, engine, clutch, tire, suspension and aerodynamic models.


W Hastings St
Waterfront Rd W
Sinclair Centre
Waterfront Station
Harbour Centre Tower
The Landing
Water St
Trounce Alley
Pacific Centre
Pacific Centre
Seymour St
W Pender St
Vancouver Art Gallery
Robson Square
Vancouver Courthouse
Library Square
Nelson Park
St Paul's Hospital
BC Place Stadium
Plaza of Nations
Coopers Park
Marinaside Crescent
Pacific Blvd
Beach Crescent
Beach Ave
Burrard St
Thurlow St
Robson St
Georgia St
Dunsmuir St
Hornby St
Howe St
Richards St
Homer St
Hamilton St
Cambie St
Beatty St
Smithe St
Nelson St
Davie St
Drake St
Helmcken St
Granville St
Mainland St
Pendrell St
Comox St
Barclay St
Haro St
Alberni St
Bute St
Jervis St
Nicola St
Broughton St
Burnaby St
Harwood St
Pacific St
Abbott St
Griffiths Way
Terry Fox Way
Expo Blvd
Mainland St
One Way
7A
99A
99
George
David

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Brochu, E., Cora, V. M. & De Freitas, N. (2010), 'A tutorial on bayesian optimization of expensive cost functions, with application to active user modeling and hierarchical reinforcement learning', *arXiv preprint arXiv:1012.2599* .

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