SOLUTIONS TO EXERCISE 3

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7.10

a & b. Consider the model

$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$$

If you multiply X_{2i} by 2, you can verify from Equations (7.4.7) and (7.4.8), that the slope for X_2 is half of its original value, and the slope for X_3 remain unaffected. On the other hand, if you multiply Y_i by 2, the slopes as well as the intercept coefficients and their standard errors are all multiplied by 2. You can compare this result with question 3.14 to see the differences.

7.14

a. As discussed in Sec. 6.9, to use the classical normal linear regression model (CNLRM), we must assume that $\ln u_i \sim N(0, \sigma^2)$ After estimating the Cobb-Douglas model, obtain the residuals and subject them to normality test, such as the Jarque-Bera test.

b. No. As discussed in Sec. 6.9, $u_i \sim \log -\text{normal}(e^{\sigma^2/2}, e^{\sigma^2}(e^{\sigma^2} - 1))$

7.20

a. Ceteris paribus, on average, a 1% increase in the unemployment rate leads to a 0.34% increase in the quite rate, a 1% increase in the percentage of employees under 25 leads to a 1.22% increase in the quite rate, and 1% increase in the relative manufacturing employment leads to 1.22 % increase in the quite rate, a 1% increase in the percentage of women employees leads to a 0.80 % increase in the quite rate, and that over the time period under study, the quite rate declined at the rate of 0.54% per year.

b. Yes, quite rate and the unemployment rate are expected to be negatively related.

c. As more people under the age of 25 are hired, the quite rate is expected to go up because of turnover among younger workers.

d. The decline rate is 0.54%. As working conditions and pensions benefits have increased over time, the quit rate has probably declined.

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e. No. Low is a relative term.

f. Since the t values are given, we can easily compute the standard errors. Under the null hypothesis that the true β_i is zero, we have the relationship:

$$t = \frac{\hat{\beta}_i}{se\left(\hat{\beta}_i\right)} \Rightarrow se\left(\hat{\beta}_i\right) = \frac{\hat{\beta}_i}{t}$$

8.2

From Eq(8.4.16)

$$F = \frac{\left(ESS_{new} - ESS_{old}\right)/NR}{RSS_{new}/\left(n-k\right)}$$

where NR = number of new regressors. Divide the numerator and denominator by TSS and recall that $R^2 = ESS/TSS$ and $(1 - R^2) = RSS/TSS$. Substituting these expressions into Eq(8.4.16), we will obtain Eq(8.4.18).

8.3

This is a definitional issue. As noted in the chapter, the unrestricted regression is known as the long, or new, regression, and the restricted regression is known as the short regression. These two differ in the number of regressors included in the models.

8.6

Start with equation Eq(8.4.11) and write it as:

$$F = \frac{\left(n-k\right)R^2}{\left(k-1\right)\left(1-R^2\right)}$$

which can be rewritten as:

$$F\frac{k-1}{n-k} = \frac{R^2}{\left(1-R^2\right)}$$

after further algebraic manipulation, we obtain

$$R^2 = \frac{\left(k-1\right)F}{\left(k-1\right)F + \left(n-k\right)}$$

which is the desired result. For regression (8.2.1), $n = 64$, $k = 3$. Therefore, $F_{.05}(2, 62) = 3.15$, approx. (Note use 60 *df* in place of 62 *df*). Therefore, putting these values in the preceding R^2 formula, we obtain:

$$R^2 = \frac{\left(3-1\right)3.15}{\left(3-1\right)3.15 + \left(64-3\right)} = 0.093$$

This is the critical R^2 value at the 5% level of significance. Since the observed of R^2 of 0.7077 in (8.2.1) far exceeds the critical value, we reject the null hypothesis that the true R^2 value is zero.

8.7

We know that

$$F = \frac{\left(R^2_{new} - R^2_{old}\right)/1}{\left(1 - R^2_{new}\right)/\left(n-k\right)} \qquad \text{by Eq. 8.5.18}$$
$$= \frac{\left(.9776 - .9388\right)/1}{\left(1 - .9776\right)/\left(20-3\right)} = 29.446$$

and we know that $F_{1,17} = t^2_{17}$. Given $\beta_3 = 23.195$

$$se\left(\hat{\beta}_3\right) = \frac{\hat{\beta}_3}{t_{17}} = \frac{\hat{\beta}_3}{\pm\sqrt{F_{1,17}}} = \frac{-23.195}{\pm\sqrt{29.446}} = \pm 4.274$$

Since $se(\hat{\beta}_3) > 0$, we obtain the standard error of X_3 is 4.274 which is quite close to the given 4.2750.

8.11

1. Unlikely, except in the case of very high multicollinearity.

2. Likely. Such cases occur frequently in applied work.

3. Likely, actually this would be an ideal situation.

4. Likely. In this situation the regression model is useless.

5. Could occur if the significance of one coefficient is insufficient to compensate for the insignificance of the other.1.

6. Unlikely

8.19

For the income elasticity, the test statistic is: $t = (0.4515 - 1)/2.2065 = 0.0247$. This t value is highly significant, refuting the hypothesis that the true elasticity is 1. For the price elasticity, the test statistic is:$t = (-0.3772 - (-1))/9.808 = 0.0635$. This t value is also significant, leading to the conclusion that the true price elasticity is different from 1.

8.20

The null hypothesis is that $\beta_2 = \beta_3$, that is, $\beta_2 + \beta_3 = 0$. Using the t statistic given in (8.6.5), we obtain:

$$t = \frac{0.4515 + (-0.3772)}{\sqrt{0.0247^2 + 0.0635^2 - 2\left(-0.0014\right)}} = 0.859$$

This t value is not significant, say at the 5% level. So, there is no reason to reject the null hypothesis.